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Quantum effects in the electron current flow in a quasi-two-dimensional electron gas

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Abstract

We consider the role of the third dimension in the conductivity of a quasi-two-dimensional electron gas (Q2DEG). If the transverse correlation radius of the scattering potential is smaller than the width of the channel, i.e. the width of the transverse electron density distribution, then virtual scattering to higher levels of the confinement potential becomes important, which causes a broadening of the current flow profile. The resulting conductivity is larger than that obtained from a quasi-classical two-dimensional Boltzmann equation. A magnetic field, parallel to the driving electric field, effectively adds strength to the confining potential. As a result, the width of the current flow profile decreases and a positive longitudinal magnetoresistivity of the Q2DEG is expected.

1. Introduction

Conductivity of a quasi-two-dimensional electron gas (Q2DEG) is usually calculated by means of a quasi-classical 2D Boltzmann equation (see, e.g., Stern and Howard 1967, Siggia and Kwok 1970, Stern 1976, Ando *et al* 1982, Cantrell and Butcher 1985, Tang and Butcher 1988a, 1988b, Das Sarma and Hwang 1999 and references therein). Quantum corrections to the 2D conductivity are assumed to be only due to the weak localization or interaction mechanisms (see, e.g., the review by Kawaji 1994). The starting point of the quasi-classical approach is the Hamiltonian

$$\hat{H} = \sum_{\alpha,k_2}\left(E_\alpha + \frac{k_2^2}{2m}\right)a^\dagger_{\alpha,k_2}a_{\alpha,k_2} + \sum_{\alpha,\beta,k_2,q_2} M_{\alpha,\beta}(q_2)a^\dagger_{\alpha,k_2+q_2/2}a_{\beta,k_2-q_2/2}, \tag{1}$$

where

$$M_{\alpha,\beta}(q_2) = \int \mathrm{d}z\, \Psi^*_\alpha(z)U(q_2,z)\Psi_\beta(z). \tag{2}$$

Here $U(q_2,z) = \int \mathrm{d}^2 r_2\, \mathrm{e}^{-\mathrm{i}q_2\cdot r_2}U(r_2,z)$ where $U(r_2,z)$ is a random scattering potential; the subscript 2 denotes here and below the 2D in-plane vectors. E_α is the αth eigenvector of the

quantum well, $V(z)$, which confines the motion of the electrons in the z-direction. $\Psi_\alpha(z)$ is the corresponding eigenfunction. A calculation of the potential $V(z)$ and the wavefunctions for a specific system may present a rather complicated quantum mechanical problem requiring a simultaneous solution of the corresponding Schrödinger and Poisson equations.

In order to derive a quasi-classical 2D Boltzmann equation, one assumes that only the highest occupied level, α_0, of the potential $V(z)$ is of importance and all transitions, both real and virtual, to lower or higher levels can be discarded. If λ_z characterizes the width of the wavefunction $\Psi_{\alpha_0}(z)$, then the characteristic energy interval between the state E_{α_0} and the neighbouring states can be estimated as $\Delta E_{\alpha_0} \approx \frac{\hbar^2}{2m\lambda_z^2}$. Then the above assumption for the real transitions is justified if the channel is narrow enough and the temperature is low enough, $\Delta E_{\alpha_0} \gg k_B T$.

Virtual transitions to other states may be caused by the random scattering potential $U(\boldsymbol{q}_2, z)$. Its off-diagonal matrix elements (2) are negligible if the scattering potential is smooth, i.e. it is characterized by a large correlation radius, $r_c \gg \lambda_z$. Then the scattering potential is responsible only for an in-plane relaxation of the electron momenta of the non-equilibrium two-dimensional electron gas (2DEG). One may then discard any possible renormalization of the current due to an admixture of states with $\alpha \neq \alpha_0$ and retain only the diagonal matrix elements $M_{\alpha_0,\alpha_0}(\boldsymbol{q})$ in equation (1). A quasi-classical 2D Boltzmann equation follows straightforwardly. If one is interested in the profile of the current flow density along the z-coordinate, it coincides in this case with the electron density profile, determined by the wavefunction of the α_0-level, i.e., $j(z) \sim \rho_{\alpha 0}(z) = |\psi_{\alpha_0}(z)|^2$. The width of this profile may be called the width of the channel.

A completely different situation arises in the opposite limit, $r_c \ll \lambda_z$, to be called below the quantum limit. The scattering potential $U(\boldsymbol{q}_2, z)$ induces strong transitions to other levels $\alpha \neq \alpha_0$ of the quantum well. The matrix elements $M_{\alpha,\beta}(\boldsymbol{q}_2)$ with $\alpha, \beta \neq \alpha_0$ cannot be discarded, and the conventional approach based on a quasi-classical 2D Boltzmann equation is not applicable. An admixture of other states with the wavefunctions, localized in much wider regions than that of $\psi_{\alpha_0}(z)$, may lead to a much broader z-profile of the current flow density. Its decay along the z-axis is characterized by a length b, which may substantially exceed the width λ_z.

The conductivity of a Q2DEG may now be sensitive to an external in-plane magnetic field, which influences the effective λ_z-value and leads to a decrease of the effective width of the channel, through which the current flows. As a result, a positive magnetoresistivity in an in-plane magnetic field is expected. It is worthwhile to distinguish this mechanism of the longitudinal magnetoresistivity from the other mechanism recently proposed by Dolgopolov and Gold (2000), which considers a 2D system without taking any account of the third dimension. The magnetic field polarizes electron spins and causes a change of the Fermi energy and, hence, of the scattering time. The paper by Herbut (2001) discusses the screening of the random potential in a system close to the metal–insulator transition and strongly spin polarized by an external magnetic field. It was argued by Maekawa and Fukuyama (1980) that a finite magnetoconductance in a parallel magnetic field can be due to the Zeeman splitting effect on the weak-localization correction to the conductivity in the presence of a spin–orbit interaction or electron spin scattering by localized magnetic moments. Depending on the parameters, the sign may be either positive or negative.

It is emphasized that the mechanism that we propose here does not consider electron spins or their polarization at all. It is of crucial importance below that the system is *quasi-two-dimensional* rather than really *two-dimensional*. The role of a finite width of the 2DEG was important in the theory proposed by Altshuler and Aronov (1981) and

Dugayev and Khmelnitskii (1984) who considered a suppression of the weak-localization corrections to the conductance in an in-plane magnetic field. This mechanism leads to a positive magnetoconductance (negative magnetoresistivity). These types of correction are not considered in this paper.

Generally, the influence of the third dimension on various physical properties of quasi-2D systems has been addressed many times. We can mention here the paper by Jonson (1976) who studied many-body effects and the applicability of various approximations to their calculation. The effect of an in-plane magnetic field on interface excitations in the finite-width Q2DEG was investigated in the series of papers by Kushwaha (1987a, 1987b, 1989).

2. The kinetic equation

A calculation of the conductivity of a Q2DEG in the quantum limit $\lambda_z \gg r_c$ cannot be carried out within the framework of a conventional quasi-classical 2D Boltzmann equation. A quantum approach is necessary, which takes into account the renormalization of all relevant quantities due to an admixture of various levels, $\alpha \neq \alpha_0$, of the quantum well. We shall see below that this is equivalent to explicitly accounting for the z-dependence of the current density flow. This analysis can best be carried out using a quantum kinetic equation for the Wigner function. A detailed discussion of a gauge-invariant derivation and an analysis of such an equation is presented in our papers (Fleurov and Kozlov 1978, Levanda and Fleurov 1994, 2001). A discussion of quantum kinetic equations can be found in the books by Mahan (1990), and Haug and Jauho (1997), which also present introductions to the diagrammatic technique, proposed originally by Keldysh (1965).

In our paper (Levanda and Fleurov 2001) two equations were derived, equations (30) and (31), which have a gauge-invariant form and govern the kinetics of an electron gas in an external electromagnetic field. Both of these equations are written in terms of the Keldysh matrix Green functions and formally constitute eight equations for eight complex functions. However, only two equations for two functions are independent and they read

$$\left[\not{\epsilon} - \frac{1}{2m}\not{\boldsymbol{P}}^2 + \frac{\hbar^2}{8m}(\not{\boldsymbol{\nabla}}^R)^2\right] G^r(p, R) = 1 + [G^r(p, R), \Sigma^r(p, R)]_+ \tag{3}$$

and

$$\begin{aligned}\left[\not{\partial}^T + \frac{1}{2m}(\not{\boldsymbol{P}} \cdot \not{\boldsymbol{\nabla}}^R + \not{\boldsymbol{\nabla}}^R \cdot \not{\boldsymbol{P}})\right] G^K(p, R) \\ = 2\{\mathrm{i}[G^c(p, R), \Sigma^c(p, R)]_- - \mathrm{i}[\tilde{G}^c(p, R), \tilde{\Sigma}^c(p, R)]_- \\ + [G^<(p, R), \Sigma^>(p, R)]_+ - [G^>(p, R), \Sigma^<(p, R)]_+\}.\end{aligned} \tag{4}$$

where the following definitions are introduced:

$$\not{\epsilon} = \varepsilon - \frac{\hbar}{2} e j_1\left(\frac{\Delta}{2}\right) \boldsymbol{E}(R) \cdot \boldsymbol{\nabla}^p;$$

$$\not{\boldsymbol{P}} = \boldsymbol{p} + \frac{\hbar}{2} e j_1\left(\frac{\Delta}{2}\right)\left(\frac{1}{c}\boldsymbol{B}(R) \times \boldsymbol{\nabla}^p + \boldsymbol{E}(R)\partial^\varepsilon\right);$$

$$\not{\partial}^T = \partial^T + e j_0\left(\frac{\Delta}{2}\right) \boldsymbol{E}(R) \cdot \boldsymbol{\nabla}^p;$$

$$\not{\boldsymbol{\nabla}}^R = \boldsymbol{\nabla}^R + e j_0\left(\frac{\Delta}{2}\right)\left(\frac{1}{c}\boldsymbol{B}(R) \times \boldsymbol{\nabla}^p + \boldsymbol{E}(R)\partial^\varepsilon\right);$$

$$j_0(x) = \sin(x)/x \qquad \text{and} \qquad j_1(x) = \sin(x)/x^2 - \cos(x)/x.$$

We use four-coordinates $R = (\boldsymbol{R}, T)$ and four-momenta $p = (\boldsymbol{p}, \varepsilon)$. The derivative with respect to R in the operator $\Delta = \hbar\partial^p\partial^R$ acts only on the fields $\boldsymbol{E}(R)$ and $\boldsymbol{B}(R)$. The other derivatives, in the left-hand sides, with respect to R and p act on all the functions.

It is emphasized that the brackets in the right-hand side of equations (3) and (4) stand *not* for commutators but rather for four-dimensional generalizations of the Moyal (sine) brackets (Moyal 1949),

$$[f(p,R), g(p,R)]_- = \frac{1}{2\mathrm{i}}\{f(p,R)\star g(p,R) - g(p,R)\star f(p,R)\}$$
$$= f(p,R)\sin\left[\frac{\mathrm{i}\hbar}{2}\hat{\Pi}\right]g(p,R),$$

and of the Baker (cosine) brackets (Baker 1958),

$$[f(p,R), g(p,R)]_+ = \tfrac{1}{2}\{f(p,R)\star g(p,R) + g(p,R)\star f(p,R)\}$$
$$= f(p,R)\cos\left[\frac{\mathrm{i}\hbar}{2}\hat{\Pi}\right]g(p,R),$$

with the $\star$-product of two functions defined by

$$f(p,X)\star g(p,X) = f(p,X)\exp\left[\frac{\mathrm{i}\hbar}{2}\hat{\Pi}\right]g(p,X).$$

Here we have introduced the operator

$$\hat{\Pi} = \frac{\overleftarrow{\partial}}{\partial\varepsilon}\overrightarrow{\partial^T} - \overleftarrow{\partial^T}\frac{\overrightarrow{\partial}}{\partial\varepsilon} + \overleftarrow{\nabla^R}\frac{\overrightarrow{\partial}}{\partial\boldsymbol{p}} - \frac{\overleftarrow{\partial}}{\partial\boldsymbol{p}}\overrightarrow{\nabla^R},$$

which can be called a Poisson operator since the result of its action on a product of two functions is similar to the Poisson brackets of these functions. The Groenewold (1946) notation is used, according to which the left and right arrows over the differential operators indicate the action either on the left or on the right functions in the product, respectively. The shape of the above equations for Green functions in the Wigner representations and the use of the star-product indicate a direct connection of this procedure to the deformation quantization for Wigner functions. Many interesting mathematical and physical aspects of this type of quantization were recently discussed by Zachos (2002).

Equations (3) and (4) are rather complicated and certain simplifications should be made. We first start with the analysis of equation (3), which is a quantum analogue of the Hamilton–Jacobi equation.

3. The Hamilton–Jacobi equation

Equation (3) allows one to find a retarded Green function $G^r(p,R)$, determining the single-electron spectrum of the system. Its imaginary part $A(p,R) = -2\,\mathrm{Im}\,G^r(p,R)$ is often called spectral function. In the representation chosen here, this function is directly connected with the Wigner quasi-distribution function. If we neglect the external fields and scattering processes ($\Sigma^r(p,R) = 0$), and retain only the confinement potential $V(Z)$, then this equation describes the free motion of electron in the XY-plane and quantization of its Z-motion due to the confinement potential. For the case of a harmonic potential, the quantization of the electron motion in the Z-direction was analysed in our paper (Levanda and Fleurov 2001; see also the discussion in the appendix), where the gauge-invariant equation for the Wigner quasi-distribution function was found.

For any confinement potential $V(Z)$, the Wigner quasi-distribution function as well as the whole retarded function G^r decay exponentially with the growing $|Z|$. If a scattering

mechanism is introduced, which is described by a mass operator Σ^r, it is reasonable to expect Σ^r to also decay exponentially with $|Z|$. Then the product $G^r\Sigma^r$ in the right-hand side of equation (3) decays much more strongly than G^r, and hence the renormalization of the Wigner quasi-distribution function due to scattering processes is negligible at large Z. It is also worth noting that the correction to G^r due to a driving in-plane electric field $\boldsymbol{E}_2$ can only be of second or higher even order. This feature was also outlined by Fleurov and Kozlov (1978). All these corrections will be neglected in the next section, when solving the kinetic equation in the linear approximation.

We will consider here the conductivity of a Q2DEG in which the electron motion in the Z-direction is confined by a potential $V(Z)$. We assume that electrons can move freely in the XY-plane and are scattered by a random potential, which is on average homogeneous and isotropic over the whole three-dimensional space. The characteristic scale of its fluctuations can be neglected as compared to any scale important for the motion in the XY-plane. However, following the discussion in the introduction, we will account for the finite scale of these fluctuations when considering the Z-motion of the electrons. Then

$$\langle U(\boldsymbol{r}_2, Z)U(\boldsymbol{r}_2', z')\rangle = \bar{U^2}\delta(\boldsymbol{r}_2 - \boldsymbol{r}_2')l(z - z'). \tag{5}$$

The subscript 2 denotes, here and below, quantities related to the XY-plane. $\bar{U^2}$ is the mean square fluctuation of the scattering potential. If the latter is due to short-range defects with a concentration c, then $\bar{U^2} = cU_d^2$ where U_d is the scattering potential of a defect. $l(z - z')$ is the normalized correlation function for the scattering potential.

Now one can write the mass operator accounting for the s-scattering (in the XY-plane) by the potential (5) as

$$\Sigma(\varepsilon; Z, p_z) = \frac{\bar{U^2}}{(2\pi\hbar)^3}\int \mathrm{d}^2 p_2\, \mathrm{d}p_z'\, G(\boldsymbol{p}_2, \varepsilon; Z, p_z')\tilde{l}(p_z - p_z'). \tag{6}$$

where $\tilde{l}(p_z)$ is the Fourier transform of the correlation function $l(z - z')$. $\boldsymbol{p}_2$ is the two-dimensional momentum in the XY-plane. The superscripts r, $>$, and K on the mass operator Σ and Green function G in the Keldysh (1965) representation are suppressed here, since equation (6) holds for all of them.

The renormalization of the Green function G in equation (6) due to scattering can be disregarded. The non-equilibrium correction to the Green function $G(\boldsymbol{p}_2, \varepsilon; Z, p_z')$ in the linear approximation is proportional to $\boldsymbol{p}_2 \cdot \boldsymbol{E}_2$, and its contribution to the mass operator (6) becomes also zero in the s-scattering case after integration over $\boldsymbol{p}_2$.

In the case of elastic s-scattering, the imaginary part of the mass operator (6),

$$\Gamma(Z, p_z; \varepsilon) = \mathrm{i}[\Sigma^{(0)<}(Z, p_z, \varepsilon) - \Sigma^{(0)>}(Z, p_z, \varepsilon)],$$

does not depend on the electron momentum $\boldsymbol{p}_2$ in the XY-plane. The equilibrium Green function in (6) depends on the difference $\varepsilon - \varepsilon_{p_2}$, with $\varepsilon_{p_2} = \frac{p_z^2}{2m}$. Then, accounting for the integration over ε_{p_2} in (6), we may also discard its dependence on ε and assume that

$$\Gamma(Z, p_z) \approx \frac{\hbar}{\tau_2 l(0)}\int \frac{\mathrm{d}p_z'}{2\pi\hbar}\,\rho_0(Z, p_z')\tilde{l}(p_z - p_z') \tag{7}$$

where $\rho_0(Z, p_z)$ is the equilibrium Wigner function describing the electron motion in the Z-direction. The Wigner function $\rho(Z, p_z)$ (at equilibrium or not) is connected with the Keldysh functions

$$\rho(Z, p_z) = \mathrm{i}\frac{1}{\gamma}\int \frac{\mathrm{d}\varepsilon\,\mathrm{d}^2 p_2}{(2\pi\hbar)^3}\,[G^<((Z, p_z; \boldsymbol{p}_2, \varepsilon)) - G^>(Z, p_z; \boldsymbol{p}_2, \varepsilon)] \tag{8}$$

where the factor γ is determined by the normalization condition

$$\int \mathrm{d}Z \frac{\mathrm{d}p_z}{2\pi\hbar} \rho(Z, p_z) = 1.$$

Equation (7), together with equations (6) and (8), defines the time τ_2 which characterizes the electron scattering in the XY-plane. The factor $l(0)$, which is necessary here from dimensional considerations, is introduced in such a way that the time τ_2 corresponds to the scattering time in the XY-plane obtained with the quasi-classical Boltzmann equation.

4. The quantum Boltzmann kinetic equation; $B = 0$

The quantum Boltzmann kinetic equation (4) has a rather complicated shape and should be simplified before we try to solve it. The model that we consider here assumes that there are two fields acting on the electrons. These are a constant electric field $\boldsymbol{E}_2$ lying in the XY-plane, and an electric field $\boldsymbol{E}_z = -\frac{1}{e}\frac{\mathrm{d}V(Z)}{\mathrm{d}Z}$ due to the confinement potential $V(Z)$. The possible role of a magnetic field will be discussed later.

Even now the equation looks too complicated. Further simplifications can be made, if we take into account that our aim here is to calculate the electric current flowing in the XY-plane. It can be expressed by means of a conditional moment of the Green function:

$$j_2(Z) = 2e\mathrm{i} \int \frac{\mathrm{d}\varepsilon}{2\pi\hbar} \int \frac{\mathrm{d}^3 p}{(2\pi\hbar)^3} \frac{\boldsymbol{p}_2}{m} \delta G^<(\boldsymbol{p}_2, \varepsilon; p_z, Z) \tag{9}$$

where $\delta G^<(\boldsymbol{p}_2, \varepsilon; p_z, Z)$ is the non-equilibrium part of the Green function $G^<(\boldsymbol{p}_2, \varepsilon; p_z, Z)$. A more detailed discussion of the properties of the conditional moments can be found in our papers (Levanda and Fleurov 1994, 2001 and references therein).

Calculating various space conditional moments of the quantum kinetic equation (4), one obtains an infinite set of equations, which yield the so-called hydrodynamic formulation of the problem (for details and relevant references, see Levanda and Fleurov 2001). We restrict ourselves to the s-scattering in the XY-plane, which implies that the equations for the components of the kinetic momentum are decoupled from the equations for higher powers of the electron momentum. Three equations, corresponding to the three components of the vector $\boldsymbol{p}$, are obtained from equation (4) by multiplying it by $\boldsymbol{p}$, and integrating its left- and right-hand sides over the variables ε and $\boldsymbol{p}$.

The equilibrium part of the resulting equation for the Z-component provides information on the quantization of motion in the Z-direction, which can also be obtained from equation (3). The non-equilibrium part is trivial with zeros in both the left- and right-hand sides, which corresponds to the absence of a current in the Z-direction. The equations for the X- and Y-components read

$$e\boldsymbol{E}_2 n_2 \bar{\rho}(Z) = 2 \int \frac{\mathrm{d}\varepsilon}{2\pi\hbar} \int \frac{\mathrm{d}^2 p_2}{(2\pi\hbar)^2} \boldsymbol{p}_2 \cos\left\{\mathrm{i}e\hbar\boldsymbol{E}_2\left[\overleftarrow{\partial^\varepsilon}\overrightarrow{\partial_2^p} - \overleftarrow{\partial_2^p}\overrightarrow{\partial^\varepsilon}\right]\right\} I_z \tag{10}$$

where

$$I_z = \int \frac{\mathrm{d}p_z}{2\pi\hbar} \left(G^<(Z, p_z; \boldsymbol{p}_2, \varepsilon)\hat{M}\Sigma^>(Z, p_z; \boldsymbol{p}_2, \varepsilon)\right.$$
$$\left. - G^>(Z, p_z; \boldsymbol{p}_2, \varepsilon)\hat{M}\Sigma^<(Z, p_z; \boldsymbol{p}_2, \varepsilon)\right) \tag{11}$$

with

$$\hat{M} = \cos\left\{\frac{\hbar}{2}\left(\overleftarrow{\partial^Z}\overrightarrow{\partial^{p_z}} - \overleftarrow{\partial^{p_z}}\overrightarrow{\partial^Z}\right)\right\}$$

and

$$\bar{\rho}(Z) = \int \frac{\mathrm{d}p_z}{2\pi\hbar}\,\rho(Z,\,p_z).$$

We have kept only the terms due to the Baker cosine brackets in the right-hand side of the kinetic equation (10) and neglected the terms due to the Moyal sine brackets. The reasons for taking this approximation may be outlined as follows. The Moyal brackets contain only terms in odd powers of the fields. Therefore, considering the equation for the Z-component we may obtain only a small correction to the equilibrium Wigner function $\rho_0(Z,\,p_z)$ due to the random scattering potential. These are the same corrections as were neglected in the previous section. As for the XY-components, the contribution due to the Moyal brackets results in a small renormalization of the driving term in the left-hand side of the quantum kinetic equation. This type of renormalization was actually calculated by Fleurov and Kozlov (1978), who called it the non-local correction to the driving term. This renormalization is proportional to a typical scattering potential value, e.g. the concentration of defects, and can be neglected here.

We analyse here equation (10) in the linear-in-$\boldsymbol{E}_2$ approximation, meaning that $\cos\bigl\{\mathrm{i}e\hbar\boldsymbol{E}_2\bigl[\overleftarrow{\partial^{\varepsilon}}\,\overrightarrow{\partial_2^{p}} - \overleftarrow{\partial_2^{p}}\,\overrightarrow{\partial^{\varepsilon}}\bigr]\bigr\} \approx 1$. Using now equations (6)–(8) we write

$$\begin{aligned} I_z &= \mathrm{i}\int \frac{\mathrm{d}p_z}{2\pi\hbar}\,\delta G(Z,\,p_z;\,\boldsymbol{p}_2,\,\varepsilon)\,\hat{M}\Gamma(Z,\,p_z) \\ &= \frac{\mathrm{i}}{\tau_2}\int \frac{\mathrm{d}p_z}{2\pi\hbar}\int \frac{\mathrm{d}p_z'}{2\pi\hbar}\,\delta G(Z,\,p_z;\,\boldsymbol{p}_2,\,\varepsilon)\,\hat{M}\rho_0(Z,\,p_z')\tilde{l}(p_z - p_z') \end{aligned} \tag{12}$$

where $\delta G(Z,\,p_z;\,\boldsymbol{p}_2,\,\varepsilon) = \delta G^{>}(Z,\,p_z;\,\boldsymbol{p}_2,\,\varepsilon) = \delta G^{<}(Z,\,p_z;\,\boldsymbol{p}_2,\,\varepsilon)$ is the non-equilibrium part of the Keldysh functions. The fact that the non-equilibrium corrections to all four Keldysh functions (including G^c and $\tilde{G}^c$) coincide follows from the principle of detailed balance, which for the quantum collision integral was proved by Fleurov and Kozlov (1978).

We also assume that the motion in the Z-direction may be separated from that in the XY-plane, meaning that

$$\delta G(Z,\,p_z;\,\boldsymbol{p}_2,\,\varepsilon) = \delta G_2(\boldsymbol{p}_2,\,\varepsilon)\,\rho_{cur}(Z,\,p_z)$$

where the function $\rho_{cur}(Z,\,p_z)$, normalized to one, will be found from the kinetic equation. This assumption allows us to rewrite equations (10) and (12) in the form

$$e\boldsymbol{E}_2 n_2\bar{\rho}_0(Z) = \frac{2}{\tau_2 l(0)}\mathcal{P}_2\int \frac{\mathrm{d}p_z}{2\pi\hbar}\int \frac{\mathrm{d}p_z'}{2\pi\hbar}\,\rho_{cur}(Z,\,p_z)\hat{M}\rho_0(Z,\,p_z')\tilde{l}(p_z - p_z') \tag{13}$$

where

$$\mathcal{P}_2 = \mathrm{i}\int \frac{\mathrm{d}\varepsilon}{2\pi\hbar}\int \frac{\mathrm{d}^2 p_2}{(2\pi\hbar)^2}\,\boldsymbol{p}_2\,\delta G_2(\boldsymbol{p}_2,\,\varepsilon).$$

Integrating by parts in (13) and carrying out Fourier transformations, the equation can be represented in the form

$$\begin{aligned} e\boldsymbol{E}_2 n_2\bar{\rho}_0(Z) = \frac{2}{\tau_2 l(0)}\mathcal{P}_2\int \frac{\mathrm{d}p_z}{2\pi\hbar}\int \frac{\mathrm{d}p_z'}{2\pi\hbar}\int \mathrm{d}z\,\mathrm{e}^{-\mathrm{i}(p_z - p_z')z}l(z) \\ \times\,\rho_{cur}\left(Z + \frac{1}{2}z,\,p_z\right)\rho_0\left(Z + \frac{1}{2}z,\,p_z'\right), \end{aligned} \tag{14}$$

which is an integral equation for the function $\rho_{cur}(Z,\,p_z)$.

4.1. Analysis of the kinetic equation

We can analyse equation (14) for a general shape of the confinement potential in the two limiting cases. First, we may assume that the characteristic correlation radius for the scattering potential is much smaller than the width of the confinement potential, i.e., we assume that $l(z) \approx \delta(z)$. Then equation (14) becomes

$$e\boldsymbol{E}_2 n_2 \bar{\rho}_0(Z) = \frac{2}{\tau_2 l(0)} \mathcal{P}_2 \int \frac{\mathrm{d}p_z}{2\pi\hbar} \int \frac{\mathrm{d}p_z'}{2\pi\hbar} \rho_{cur}(Z, p_z)\rho_0(Z, p_z') = \frac{2}{\tau_2 l(0)} \mathcal{P}_2 \bar{\rho}_{cur}(Z)\bar{\rho}_0(Z) \tag{15}$$

where

$$\bar{\rho}_{cur}(Z) = \int \frac{\mathrm{d}p_z'}{2\pi\hbar} \rho_{cur}(Z, p_z).$$

As follows from equation (15), the distribution $\bar{\rho}_{cur}(Z)$ does not depend on Z. The current density becomes

$$\boldsymbol{j}_2(Z) \propto \sigma_2 \boldsymbol{E}_2 \tag{16}$$

where $\sigma_2 = \frac{e^2 n_2 \tau_2}{m}$ is the Drude conductivity of the 2DEG.

The result seems to be very strange, since the current density is independent of Z. This means that the current density profile, $\bar{\rho}_{cur}(Z)$, is infinitely broad. The result for the current flow profile is virtually the same as if there were no confinement potential at all. Physically, this is connected with the fact that the scattering potential causes virtual transitions to upper energy levels of the confinement potential where the wavefunctions are broader than in the ground state. In the case of the δ-correlated scattering potential, all the levels equally contribute to $\bar{\rho}_{cur}(Z)$ and the latter becomes infinitely broad.

This scenario is extreme and certainly not realistic. First, there is no such thing as an exactly δ-correlated scattering potential; it always has a finite width. Second, this peculiar Z-independent behaviour of the non-equilibrium correction δG means that, at high enough values of the coordinate Z, δG may become larger than the value of the Green function G at equilibrium, since the latter deceases exponentially with $|Z|$. This means that the linear-in-the-driving-field $\boldsymbol{E}_2$-approximation cannot be applied for such values of Z. We shall discuss possible roles of non-linear effects below.

Now we consider the opposite limiting case, where the characteristic correlation radius for the scattering potential is much larger than the width of the confinement potential. Then the convergence of the integrals in the kinetic equation is limited by the width of the confinement potential. Therefore, we may assume that the correlation function is a constant, $l(z) \approx l(0)$, or $\tilde{l}(p) \approx 2\pi\hbar l(0)\,\delta(p)$. Then the integrand in equation (13) can be written in terms of the $\star$-product as $\rho_{cur}(Z, p_z) \star \rho_0(Z, p_z)$. The equilibrium Wigner function $\rho_0(Z, p_z)$ is a so-called $\star$-gen function of the confinement potential (see, e.g., Zachos (2002) for definitions and properties of the $\star$-gen functions). In particular they possess a property according to which

$$\rho_0(Z, p_z) \star \rho_0(Z, p_z) = 2\pi\hbar\rho_0(Z, p_z).$$

This allows us to conclude that the function $\rho_{cur}(Z, p_z) = \rho_0(Z, p_z)$ solves equation (13). As a result we get

$$\boldsymbol{j}_2(Z) = \sigma_2 \boldsymbol{E}_2 \bar{\rho}_0(Z). \tag{17}$$

Now the current density profile coincides with the equilibrium electron density profile, meaning that excitations to higher energy levels of the confinement potential are of no importance. Integrating over Z, we conclude that the two-dimensional conductivity is finite and coincides with σ_2. The result (17) can be obtained directly from the quasi-classical 2D Boltzmann equation.

4.2. The harmonic confinement potential

Here we consider the electric conductivity of a Q2DEG created by a harmonic confinement potential $V(Z) = \frac{m\omega^2 Z^2}{2}$ with the amplitude of the zero-point oscillations $u = \sqrt{\frac{\hbar}{m\omega}}$. We also assume that the correlation function for the scattering potential has a Gaussian shape:

$$l(z) = \frac{1}{r_c\sqrt{2\pi}} \exp[-z^2/2r_c^2]. \tag{18}$$

This case allows for an analytic solution over the whole range of the ratios r_c/u, from zero to infinity. The two limits of this ratio, $r_c \ll u$ and $r_c \gg u$, correspond to two limiting cases considered above.

The properties of the equilibrium gauge-invariant Wigner quasi-distribution function in a harmonic potential including a homogeneous magnetic field are described in our earlier paper (Levanda and Fleurov 2001; see also the discussion in the appendix). If we assume here that the electron Fermi level lies within the two-dimensional subband corresponding to the ground state of the harmonic confinement potential, the Z-dependent part of the equilibrium Wigner function for these electrons is

$$\rho_0(Z, p_z|u) = 4\pi\hbar u\sqrt{\pi} \exp\left\{-\frac{u^2 p_z^2}{\hbar^2}\right\} \bar{\rho}_0(Zu) \tag{19}$$

where

$$\bar{\rho}_0(Z|u) \equiv \frac{1}{u\sqrt{\pi}} \exp\left(-\frac{Z^2}{u^2}\right)$$

describes the electron density profile at equilibrium.

Now the functions (18) and (19) are substituted into (14) and we obtain an integral equation for the function $\rho_{cur}(Z, p_z)$. This function is looked for also in a Gaussian shape, i.e., we assume that $\rho_{cur}(Z, p_z) = \rho_0(Z, p_z|b)$ where the function $\rho_0(Z, p_z|b)$ is defined by equation (19), in which the amplitude of the zero-point oscillations u is replaced by an unknown parameter b. Carrying out all the necessary integrations, equation (14) becomes

$$e\boldsymbol{E}_2 n_2 \bar{\rho}_0(Z) = 2\mathcal{P}_2 \frac{\tilde{r}_c}{ub\tau_2} \exp\left\{-Z^2\left(\frac{1}{\tilde{u}^2} - \frac{\tilde{r}_c^2}{2\tilde{u}^4}\right)\right\} \tag{20}$$

where

$$\frac{1}{\tilde{u}^2} = \frac{1}{u^2} + \frac{1}{b^2}$$

and

$$\frac{1}{\tilde{r}_c^2} = \frac{1}{r_c^2} + \frac{1}{u^2} + \frac{1}{b^2}. \tag{21}$$

Now we compare the Z-dependence of the right-hand side of equation (20), and that of the left-hand side, determined by the distribution $\bar{\rho}_0(Z)$. The requirement that these dependences coincide yields the equation

$$\frac{1}{\tilde{u}^2} - \frac{\tilde{r}_c^2}{2\tilde{u}^4} = \frac{1}{u^2}$$

whose solution,

$$\frac{1}{b^2} = -\frac{1}{r_c^2} + \sqrt{\frac{1}{r_c^4} + \frac{1}{u^4}}, \tag{22}$$

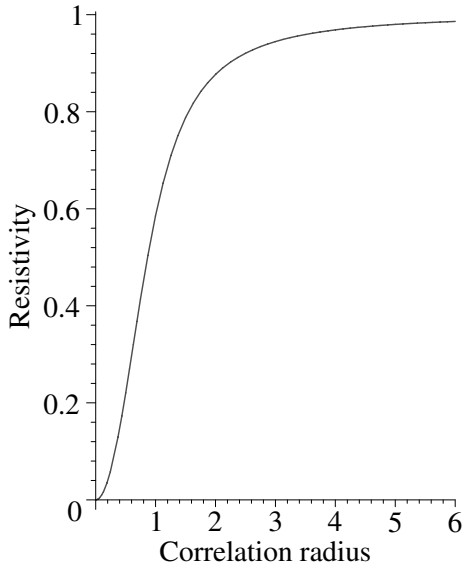

Figure 1. The dependence of the relative resistivity ϱ of the Q2DEG on the ratio of the correlation radius of the scattering potential, r_c, and the width of the channel u.

determines the scale of the current flow profile (9). b is now the effective width of the channel, which does not necessarily coincide with the width u of the electron density profile. One can readily see that $b \approx u$ only in the limit of large correlation radius $r_c \gg u$. However, it may become very large, $b \approx \frac{2u^2}{r_c} \gg u$, in the quantum limit, $r_c \ll u$. These are just the two limiting cases discussed in section 4.1.

The density profile of the electric current flow (9) in the Q2DEG becomes

$$\boldsymbol{J}(Z) = \sigma_2 \frac{1}{\sqrt{2\pi}\tilde{r}_c} \exp\left(-\frac{Z^2}{b^2}\right)\boldsymbol{E}_2 \tag{23}$$

where $\sigma_2 = \frac{e^2 n_2 \tau_2}{m}$ is the 2D Drude conductivity. Integrating equation (23) over Z one gets the conductivity of the Q2DEG, $\sigma = \sigma_2 \frac{b}{\sqrt{2}\tilde{r}_c}$, which strongly depends on the ratio of the width of the electron density profile u and the correlation radius r_c of the scattering potential. The Drude formula is applicable only if the long-range fluctuations are characteristic of the scattering potential, i.e., $r_c \gg u$; then $\frac{b}{\sqrt{2}\tilde{r}_c} \to 1$, and $\sigma = \sigma_2$. However, in the quantum limit, $r_c \ll u$, when the scale of the scattering potential fluctuations is smaller than the width of the channel, a strong deviation from the Drude formula is expected. The effective width of the channel where the current actually flows, $b \approx \frac{u^2}{r_c} \gg u$, may become much larger than the width u of the equilibrium electron density profile. As a result, the conductivity may also become much larger: $\sigma = \sigma_2 \frac{u^2}{r_c^2} \gg \sigma_2$.

This type of behaviour is illustrated in figure 1 where the relative resistivity $\varrho = \sigma_2/\sigma$ is plotted as function of the relative correlation radius r_c/u of the scattering potential. It starts from zero at small correlation radii and saturates at 1 at large radii.

4.3. Applicability of the linear approximation

The above results are obtained within the linear response approximation. In order to probe the applicability limits of this approximation, one considers the Z-profile of the drift velocity (see equation (43) in Levanda and Fleurov (2001)):

$$\boldsymbol{v}_{drift} = \frac{1}{\bar{\rho}_0(Z)} \int \frac{\mathrm{d}\varepsilon}{2\pi\hbar} \int \frac{\mathrm{d}^2 p_2}{(2\pi\hbar)^2} \frac{\boldsymbol{p}_2}{m} \delta G_2(\boldsymbol{p}_2, \varepsilon). \tag{24}$$

This definition of the drift velocity yields also the conventional equation $\boldsymbol{J}(Z) = en_0(Z)\boldsymbol{v}_{drift}(Z)$, in which $n_0(Z) = n_2\bar{\rho}_0(Z)$ is the electron density at equilibrium.

One can readily see that differing current and density profiles may lead to growth of the drift velocity with increasing Z. This indicates that however small the electric field E_2 is, there are distances that are large enough for conditions to be achieved where the linear approximation in E_2 is violated. The analysis of this non-linear problem is rather complicated and requires a proper account of various inelastic processes. However, the most important (and sufficient for our purposes) conclusions can be reached in a much simpler way.

We obtain a rough but sufficient (upper limit) criterion for the applicability of the linear approximation by requiring that the additional energy acquired by an electron due to the current flow does not exceed the temperature, i.e. $v_{drift}\, p_F \ll k_B T$ where p_F is the Fermi momentum. Hence, the linear approximation is violated if $Z > Z^*$ where

$$Z^{*2} = \frac{u^2 b^2}{b^2 - u^2} \ln\left| \frac{e\tau_2 p_F}{m} \frac{u}{\tilde{r}_c} \frac{E_2}{k_B T} \right|.$$

It is important to emphasize that although this condition is obtained for the harmonic potential well considered here, its meaning is more general. For any potential well there exists a value Z^* such that at larger distances the linear approximation does not hold. At $Z > Z^*$ one has to consider a non-linear problem involving contributions of various inelastic scattering mechanisms (say, electron–phonon interactions). Without making detailed calculations, one can understand that in this non-linear region the scattering becomes much stronger and the current becomes much smaller than what follows from its profile obtained in the linear approximation; hence, its contribution to the total current can be neglected.

When integrating the current flow density over Z, one may just cut the integration at $|Z| = Z^*$, which provides a reasonable estimate $\delta\sigma$ for the non-linear corrections to the 2D conductivity:

$$\frac{\delta\sigma}{\sigma} = -2 \int_{Z^*/b}^{\infty} \mathrm{e}^{-x^2}\, \mathrm{d}x.$$

Z^* increases with the decreasing electric field, and at $Z^* \gg b$ this correction is hardly observable.

5. Current flow in an in-plane magnetic field

It is worthwhile discussing here the influence of an in-plane magnetic field $\boldsymbol{B}$ on the conductivity. According to the intuition based on classical ideas about the electron motion, we do not expect any influence of an in-plane magnetic field, especially if it is directed *parallel* to the electric field. Nevertheless, as we demonstrate below, such an influence exists and an increase of the resistivity in a longitudinal magnetic field is expected. In the presence of a magnetic field $\boldsymbol{B}$ the kinetic equation (10), linearized with respect to $\boldsymbol{E}_2$, takes the form

$$e\boldsymbol{E}_2 n_2 \bar{\rho}_0(Z) + \frac{e}{mc} \boldsymbol{v}_{drift}(Z) \times \boldsymbol{B} = 2 \int \frac{\mathrm{d}\varepsilon}{2\pi\hbar} \int \frac{\mathrm{d}^2 p_2}{(2\pi\hbar)^2} \boldsymbol{p_2} I_z. \tag{25}$$

We will again restrict ourselves to the harmonic confinement potential and the Gaussian correlation function for the scattering potential. The choice of the equilibrium Wigner function is described in the appendix. With the proper normalization, the ground state, $n = 0$, Wigner function takes a form similar to (19):

$$\rho_0(Z, p_z|\bar{u}(B)) = 4\pi\hbar\bar{u}(B)\sqrt{\pi}\exp\left\{-\frac{\bar{u}(B)^2 p_z^2}{\hbar^2}\right\}\bar{\rho}_0(Z) \tag{26}$$

where

$$\bar{\rho}_0(Z|\bar{u}(B)) \equiv \frac{1}{\bar{u}(B)\sqrt{\pi}}\exp\left(-\frac{Z^2}{\bar{u}(B)^2}\right).$$

The only difference from (19) is that the length defined by the equation $\bar{u}^{-4}(B) = u^{-4} + l_B^{-4}$ is substituted for the zero-point amplitude u of the harmonic oscillator. This new length now characterizes the electron density profile. It is smaller than u and decreases when the magnetic length $l_B^{-2} = \frac{eB}{\hbar c}$ decreases with the growing magnetic field.

The calculation of the conductivity in an in-plane magnetic field proceeds along the same lines as in the absence of the magnetic field. We may directly use the results of section 4.2, substituting the length $\bar{u}(B)$ for u. Then the relative 2D resistivity (in units of $1/\sigma_2$) as a function of the magnetic field is

$$\varrho(B) = \frac{\sqrt{2}\tilde{r}_c(B)}{b(B)} \tag{27}$$

where

$$\frac{1}{b^2(B)} = -\frac{1}{r_c^2} + \sqrt{\frac{1}{r_c^4} + \frac{1}{\bar{u}^4(B)}}, \tag{28}$$

and

$$\frac{1}{\tilde{r}_c^2(B)} = \frac{1}{r_c^2} + \frac{1}{\bar{u}^2(B)} + \frac{1}{b(B)^2}. \tag{29}$$

The density of the electric current is now

$$\boldsymbol{J}(Z) = \sigma_2\frac{1}{\sqrt{2\pi}\tilde{r}_c(B)}\exp\left(-\frac{Z^2}{b^2(B)}\right) \tag{30}$$

and the magnetic field-dependent length $b(B)$ determines the width of the current flow profile.

The above expressions are obtained under the assumption that the magnetic field $\boldsymbol{B}$ is parallel to the driving electric field $\boldsymbol{E}_2$, when the magnetic field-dependent term in the left-hand side of equation (25) is zero. If the magnetic field still lies in the XY-plane but is not parallel to the electric field, possible corrections to the resistivity are small at least at magnetic fields that are not extremely high.

One can now study the magnetic field dependence of the resistivity (27) for various values of the ratio r_c/u. This dependence is exhibited in figure 2, which shows the variation of the relative resistivity $\varrho(B)$ as a function of the magnetic field, measured in units of $B_0 = \frac{\hbar c}{eu^2}$. The characteristic field B_0 appearing here is just the value of the magnetic field at which the magnetic length l_B becomes equal to the equilibrium width of the channel u. At $B = 0$ the curves start from various values, which become smaller as the correlation radius r_c becomes smaller. The dependence of these starting points on the correlation radius is, in fact, depicted in figure 1.

In the limit of high magnetic fields the resistivity grows and the curves tend to the common saturation level, 1. The effective width of the channel $\bar{u}(B)$ decreases with increasing magnetic

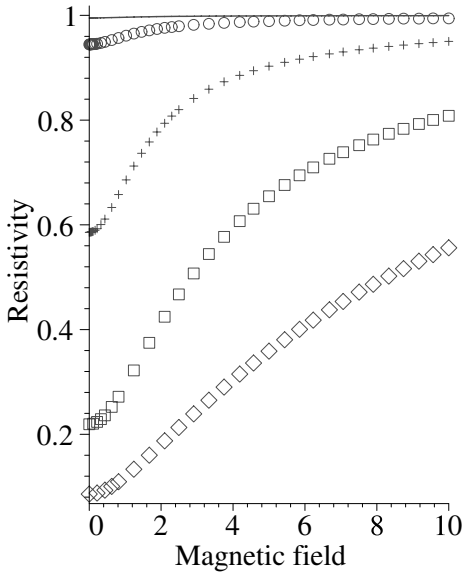

Figure 2. Relative 2D resistivity, $\varrho(B)$, as a function of the longitudinal magnetic field in units of $B_0 = \frac{\hbar c}{eu^2}$ for different ratios r_c/u: 0.3: ◇; 0.5: □; 1: ×; 3: ○; 10: — —.

field, and at a large enough magnetic field it will become smaller than the correlation radius r_c of the scattering potential. This means that the magnetic field may drive the system from the quantum ($r_c \ll \bar{u}$) to the classical ($r_c \gg \bar{u}$) limit, when the absolute resistivity acquires its Drude value $\varrho_2 = 1/\sigma_2$. This is an interesting feature of the conductivity of a Q2DEG in an in-plane magnetic field: the classical limit is reached at high rather than at low magnetic fields. Measuring the high-field limit of the resistivity yields its Drude value—the more so, since the weak-localization corrections are also suppressed by the magnetic field.

Experimentally, this limit requires very high magnetic fields. If we take a typical width of the channel to be about 100 Å, then the characteristic field B_0 is about 10 T. So high a static magnetic field is achievable in modern magnetic facilities. Pulse magnets allow one to reach even higher magnetic fields. One can also try to work with wider channels, where B_0 may be substantially smaller. However, increase in the width of the channel is limited from above by the requirement of quantization in the Z-direction.

It may be easier to carry out measurements in the low-field limit ($B < B_0$) where a positive magnetoresistivity can be expected. We can write the low-field expansion of the resistivity as

$$\frac{\varrho(B)}{\varrho(0)} = 1 + k\frac{B^2}{B_0^2} \tag{31}$$

where

$$k = \frac{r_c^2}{4\sqrt{u^4 + r_c^2}} \frac{u^2 + r_c^2 - \sqrt{u^4 + r_c^4}}{\sqrt{u^4 + r_c^4} - u^2}.$$

The dependence of this coefficient on the ratio r_c/u is shown in figure 3. It starts from $k = 0.5$ at $r_c \ll u$ and tends to 0 at $r_c \gg u$. This curve also reflects the feature of the Q2DEG resistivity in a magnetic field observed in figure 2: that the resistivity is most sensitive to the magnetic field in the quantum limit and does not depend on it in the classical limit.

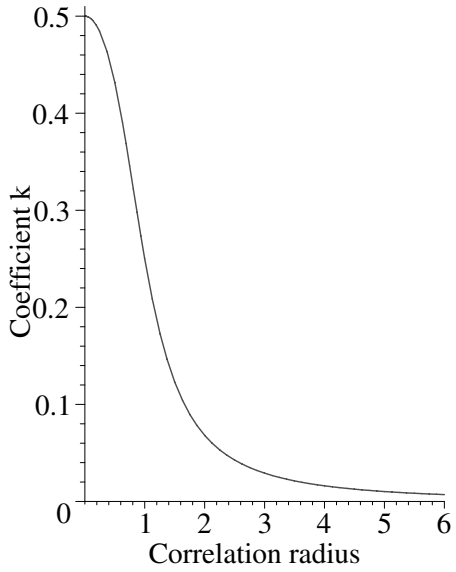

Figure 3. The dependence of the coefficient k in the low-field expansion (31) of the resistivity $\varrho(B)/\varrho(0)$ on the ratio of the correlation radius of the scattering potential, r_c, and the width of the channel u.

6. Conclusions

The results of this paper emphasize that the difference between the 2DEG and Q2DEG, however subtle it is, may be of crucial importance for the transport processes. If the scattering potential comprises fluctuations with a small enough correlation radius, the profile of the electron current may become broader than the profile of the electron density. Longitudinal positive magnetoresistivity is proposed as an experimentally observable consequence.

The formal analysis is carried out for a harmonic confinement potential and a homogeneously distributed scattering potential, characterized by a Gaussian correlation function. Then all the integro-differential kinetic equations are solvable analytically. Consideration of other shapes of the confinement and scattering potentials will require numerical analysis and may lead to various peculiar details of the behaviour of the current flow profile and the resistivity. However, we believe that the most important qualitative features outlined in this paper will hold.

A negative longitudinal magnetoresistivity observed in thin In_2O_{3-x} films by Ovadyahu *et al* (1985) was attributed to the suppression of the weak-localization correction in accordance with the theory due to Altshuler and Aronov (1981) and Dugayev and Khmelnitskii (1984). There was also an investigation of this suppression in silicon inversion layers carried out by Menz and Wheeler (1987).

A longitudinal magnetoresistivity was also observed in SiGe layers (Okamoto *et al* 1999a, 1999b). The authors attributed the effect to the electron spin polarization (a theory was presented by Dolgopolov and Gold (2000)). This explanation seems to be quite reasonable, since the electron concentrations in these experiments are so low that a complete spin polarization is possible. The mechanism proposed in this paper may become more important at a higher electron concentration. We do not know currently of any relevant experimental data and the question remains open.

Acknowledgments

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Appendix A. Appendix. The Wigner function of a harmonic oscillator in a magnetic field

As shown in Levanda and Fleurov (2001), the Wigner function of a harmonic oscillator in a magnetic field satisfies the equation

$$\left\{\varepsilon + \frac{\hbar^2}{6}m\omega^2(\partial^{p_z})^2 + \frac{\hbar^2}{4m}[\boldsymbol{\nabla}^X - m\omega^2 Z\hat{\boldsymbol{n}}_Z\partial^\varepsilon + m\omega_B\hat{\boldsymbol{n}}_B \times \boldsymbol{\nabla}^p]^2 - \frac{1}{m}\left[\boldsymbol{p} - \hat{\boldsymbol{n}}_Z\frac{\hbar^2}{12}m\omega^2\partial^{p_z}\partial^\varepsilon\right]^2\right\}\rho_W(\varepsilon, \boldsymbol{p}, \boldsymbol{R}) = 0, \tag{A.1}$$

where $\hat{\boldsymbol{n}}_z$ and $\hat{\boldsymbol{n}}_B$ are the unit vectors in the directions of the Z-axis and of the magnetic field $\boldsymbol{B}$, respectively; $\omega_B = \frac{eB}{mc}$. A set of functions solving this equation reads, in the case of $\hat{\boldsymbol{n}}_B \parallel \hat{\boldsymbol{n}}_y$,

$$\rho_W(\varepsilon, \boldsymbol{p}, Z|n) = \sqrt{\frac{m\bar{\omega}}{\pi\hbar}}\sqrt{\frac{\hbar^2}{6m\omega^2\Omega^2}}\exp\left[-\frac{m\bar{\omega}}{\hbar}\left((Z+Z_0)^2 + \frac{6\Omega^2}{m\omega^2}\right)\right] \times \cos\left(\frac{p_z}{\hbar}\sqrt{\frac{24\Omega^2}{m\omega^2}}\right)\theta(\Omega^2)L_n\left(\frac{4}{\hbar\bar{\omega}}\left(\frac{m\bar{\omega}^2}{2}(Z+Z_0)^2 + \frac{p_z^2}{2m}\right)\right) \tag{A.2}$$

where the L_n are the Laguerre polynomials, $\bar{\omega}^2 = \omega^2 + \omega_B^2$, $\theta(x)$ is the Heavyside step function. Ω is defined by means of the equation

$$0 = \Omega^2 + \varepsilon - \left(\frac{1}{2} + n\right)\hbar\bar{\omega} + \frac{1}{2}m\omega^2 z^2 - \frac{p_y^2}{2m} - \frac{1}{2}m\bar{\omega}^2 Z_0^2,$$

and

$$Z_0 = \frac{(p_x - m\omega_B Z)\omega_B}{m\bar{\omega}^2}. \tag{A.3}$$

The Wigner functions (A.2) integrated over the variable ε which are of most interest to us here are

$$\rho_W(P, z|n, p_x) = \exp\left[-\frac{2}{\hbar\sqrt{\omega_c^2 + \omega_H^2}}\left(\frac{m\bar{\omega}^2}{2}(z+Z_0)^2 + \frac{p_z^2}{2m}\right)\right] \times L_n\left(\frac{4}{\hbar\bar{\omega}}\left(\frac{m\bar{\omega}^2}{2}(Z+Z_0)^2 + \frac{p_z^2}{2m}\right)\right). \tag{A.4}$$

It is important to emphasize that the Z-dependence appears only via the combined term Z_0, equation (A.3). This means that the coordinate origin along the Z-axis is actually not defined. Its shift is equivalent to a corresponding shift of the quantum number p_x, which is the electron momentum in the direction perpendicular to the magnetic field. One can follow this degeneracy directly from equation (A.1). Any function f of the variable Z_0 commutes with the operator acting in the left-hand side of this equation on the Wigner function. Therefore, multiplying Wigner functions, whether integrated over ε, equation (A.4), or not, equation (A.2), by any function $f(Z_0)$, one again gets functions satisfying equation (A.1) or its version integrated over ε. This type of degeneracy holds only for quadratic potentials. Coming back to equation (3), from which (A.1) has been deduced, one sees that terms that are quartic or of higher order in Z in the confinement potential would result in terms depending on Z not necessarily via the

combination Z_0. These terms will lift the degeneracy and fix the zero on the Z-axis. This degeneracy also does not exist for exactly zero magnetic field, but appears for any (however small) non-zero magnetic field.

In the case of a harmonic oscillator and a homogeneous magnetic field we should take care of fixing the zero from physical considerations. It is quite clear that the preferable choice is that the minimum of the confinement potential lies at $Z = 0$. This requirement is fulfilled if $f(Z_0) = \delta(p_x - m\omega_B Z)$. Then we may write the integrated-over-ε Wigner function (A.4) in the form

$$\rho_W(P, z|n, p_x) = \delta(p_x - m\omega_B Z) \exp\left[-\left(\frac{Z^2}{\bar{u}^2} + \frac{\bar{u}^2 p_z^2}{\hbar^2}\right)\right] L_n\left(2\left(\frac{Z^2}{\bar{u}^2} + \frac{\bar{u}^2 p_z^2}{\hbar^2}\right)\right), \tag{A.5}$$

where $\bar{u}^{-4} = u^{-4} + l_B^{-4}$, and $l_B^{-2} = \frac{eB}{\hbar c}$ is the magnetic length. The same result can be obtained if the eigenfunctions for this problem are found and then substituted into the definition of the gauge-invariant Wigner function (equation (16) in Levanda and Fleurov 2001).

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